PHYSICAL REVIEW B **77**, 081301(R) (2008)

Activation gaps of fractional quantum Hall effect in the second Landau level

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(Received 2 July 2007; revised manuscript received 20 December 2007; published 1 February 2008)

We present activation gap measurements of the fractional quantum Hall effect (FQHE) in the second Landau level. Signatures for 14 (5) distinct incompressible FQHE states are seen in a high- (low-) mobility sample with the enigmatic 5/2 even-denominator FQHE having a large activation gap of ~500 (~250) mK in the high- (low-) mobility sample. This is the largest gap ever reported for the 5/2 FQHE state. Our measured large relative gaps for 5/2, 7/3, and 8/3 FQHEs indicate the emergence of exotic FQHE correlations in the second Landau level, possibly different from the well-known lowest-Landau-level Laughlin correlations. Our measured 5/2 gap is found to be in reasonable agreement with the theoretical gap once finite-width and disorder-broadening corrections are taken into account.

DOI: 10.1103/PhysRevB.77.081301 PACS number(s): 73.43.−f, 73.21.Fg

The clean (i.e., high-mobility) two-dimensional electron system (2DES) at low temperatures and high magnetic fields exhibits a rich array of exotic, highly correlated incompressible ground states. In the lowest Landau level (LLL), the physics is dominated by the sequence of fractional quantum Hall effect (FQHE) states at odd denominator filling fractions with more than 50 FQHE states with odd denominators as large as 19 observed so far. The Laughlin wave function describes the primary FQHE states at LLL filling fractions $\nu=1/m$, $m=3,5,7,\ldots$, as an incompressible quantum fluid of electrons.[1] The Laughlin states feature a gap in the energy spectrum with fractionally charged quasiparticles with charge $q=\pm e/m$ as the lowest-energy excitation. The sequence of hierarchical $\nu=p/(2p\pm 1)$, $p=1,2,3,\ldots$, higher-order FQHE states is described by the composite fermion model.[2,3] The odd-denominator constraint arises from the antisymmetry of the many-body wave functions required under the exchange of two electrons.[1] To date no even-denominator FQHE state has been observed in the LLL for a single-layered 2DES, although certain anomalies have been observed at $\nu=3/8$.[4]

The startling exception to the odd-denominator rule is the even-denominator FQHE at $\nu=5/2=(2+1/2)$ in the second Landau level (SLL). Early experiments[5] showed a weakly formed quantized Hall plateau with a finite longitudinal resistance at low temperatures. Improvement in the sample quality led to the formation of a fully quantized Hall plateau along with a vanishing longitudinal resistance at low temperatures.[6–8] In contrast to the LLL, the SLL features an array of competing ground states including odd-denominator FQHE, reentrant insulating states, and even-denominator FQHE at $\nu=5/2$ and $19/8=(2+3/8)$.[7]

The theoretical understanding of the even-denominator FQHE at $\nu=5/2$ is based on the p-wave pairing of composite fermions, similar to the pairing in a chiral p-wave BCS superconductor.[9,10] The variational wave function for the paired Hall states is modified by a Pfaffian that creates a FQHE state. Numerical diagonalization calculations provide support for the Pfaffian state as the ground state at $\nu=5/2$.[11,12] The non-Abelian quasiparticle statistics of the Pfaffian 5/2 state has received much attention recently for the prospect of realizing topologically protected qubits.[13] The existence of the even-denominator FQHE state and the general paucity of a large number of odd-denominator fractions clearly differentiate the FQHE physics of the SLL from that in the LLL. In particular, as we demonstrate in this paper, the standard composite fermion LLL hierarchy states seem to be strongly suppressed in the SLL. The nature of SLL interaction and correlation are not well understood, and the 5/2 state, although it is an even denominator state with no analogy in the LLL, is both the best understood and strongest FQHE state in the SLL. In fact, theoretical work[14] indicates that only $\nu<2+1/3$ Laughlin states would be stable in the SLL.

In this paper, we report on the observation of a large (~14) number of possible incompressible states and their activation energy gaps in the second Landau level. We find that the energy gap of the $\nu=5/2$ FQHE states exceeds 500 mK in the high-mobility sample. This is the largest measured energy gap ever reported for the 5/2 state. Comparing results from two samples with "high" and "low" mobilities, we conclude that in general the $\nu=5/2$ state is the most robust FQHE state in the SLL. The fact that an even-denominator fraction, considered to be a p-wave paired Hall state, is one of the strongest FQHE states in the SLL provides a sharp contrast between the FQHE physics in the LLL and SLL. We emphasize that our work reports the very first systematic measurement of the FQHE activation gaps in the SLL. In particular, the measured gaps for 11/5, 14/5, and 16/7 have not been reported earlier in the literature.

Two symmetrically δ-doped 30-nm-wide quantum well samples with identical structures were studied. The mobility for sample A (high-mobility) is $\mu=28.3\times 10^6$ cm^2/(V s) with an electron density of $n=3.2\times 10^{11}$ cm^{-2}. The mobility for sample B (low-mobility) is $\mu=10.5\times 10^6$ cm^2/(V s) with an electron density of $n=2.8\times 10^{11}$ cm^{-2}. The samples in a van der Pauw geometry were attached to the cold finger of a dilution refrigerator. Measurement was made using a low-frequency ac lock-in technique at low temperatures after illuminating the specimens with a red light emitting diode at 4 K.

Figures 1(a) and 1(b) show the low-temperature magnetoresistance for sample A and sample B. In the higher-mobility sample A, a remarkable array of 14 different FQHE

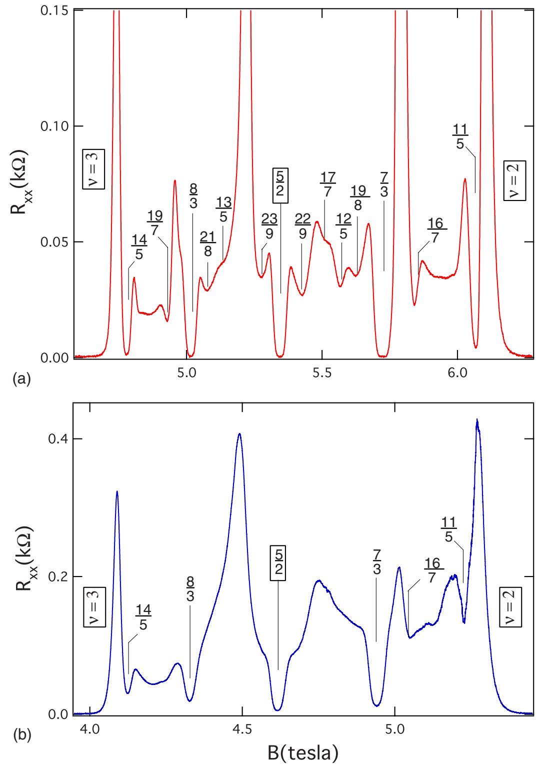

FIG. 1. (Color online) Magnetoresistance in the second Landau level of a two-dimensional electron system with (a) mobility of $\mu=28\times10^6$ cm^2/(V s) (sample *A*) and (b) mobility of $\mu=10.5\times10^6$ cm^2/(V s). The temperature was 36 mK for both samples.

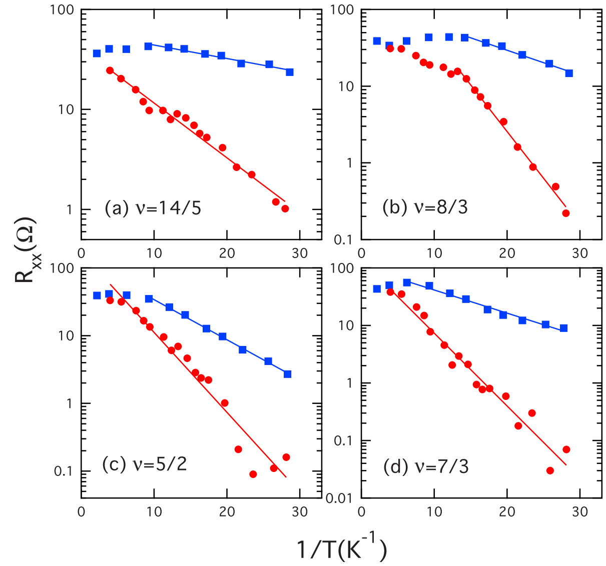

FIG. 2. (Color online) Temperature dependence of R_{xx} for various fractions in the second Landau level for samples *A* (solid circle) and *B* (solid square).

states are observed, as reflected in well-defined ρ_{xx} minima even at a relatively moderate temperature of 36 mK. Although some of the FQHE and insulating states are weaker compared to the data at lower temperature (9 mK) in Ref. 7, most states previously observed appear in sample *A*. The most prominent FQHE states are found at fillings $\nu=5/2$, 7/3, 8/3, 14/5, 11/5, 12/5, 16/7, and 19/7. Additional features may be attributed to $\nu=13/5$, 17/7, 22/9, and 23/9 states.

Figure 1(b) shows that only five of the 14 states seen in sample *A* survive in sample *B*: the FQHE states at $\nu=5/2$, 7/3, 8/3, 11/5, and 14/5. This is presumably a direct suppression due to disorder since the two samples are very similar except for a factor of ~3 difference in mobility. A cursory look at our Fig. 1 immediately makes it obvious that the strongest SLL FQHE states occur at $\nu=5/2$, 7/3, and 8/3 with the 5/2 and the 7/3 states being comparable and the 8/3 state being somewhat weaker.

Figures 2(a)–2(d) show the temperature dependence of the magnetoresistance mimina at filling fractions $\nu=14/5$, 8/3, 5/2, and 7/3, respectively, for samples *A* and *B*. Except for the highest temperatures where activated behavior is not expected, the Arrhenius plot shows that the regions of activation extend well over one decade in sample *A* for $\nu=14/5$, 8/3, 5/2, and 7/3. The energy gap Δ for the various FQHE states can be determined from the standard Arrhenius analysis using the activated resistance $R_{xx}\propto\exp(-\Delta/2T)$. A least-squares fit was performed below the high-temperature saturation point over the range of temperatures over which the resistance is activated. When the activated range is less than a decade but data scatter is not overwhelming, we show the fit values as the upper limit of the energy gap (e.g., 8/3, 14/5 states in Fig. 2). The results of our activation analysis are summarized in Table I. The energy gaps for $\nu=8/3$, 5/2, and 7/3 in sample *A* are the largest with their magnitudes exceeding 500 mK. In sample *B* the 5/2 FQHE state is by far the strongest incompressible state with the largest energy gap. Although the activated resistance is expected to saturate when the temperature $\sim\Delta/2$, the origin of this early saturation for 8/3 state is not clear at this moment. It may be related to the anomalous angular dependence of the 7/3 and 8/3 states.[15]

Figure 3 shows the energy gaps for 5/2, 7/3, 8/3, 11/5, and 14/5 states measured in a single sample. Also, energy gaps of 5/2, 7/3, and 8/3 states of the two different mobilities are compared. The gap magnitudes have been converted to the units of Coulomb energy $e^2/\epsilon\ell$, where $\epsilon=13.1$ is the background dielectric constant for GaAs and $\ell=\sqrt{\hbar/eB}$ is the magnetic length. The gaps for the $\nu=8/3$, 5/2, and 7/3 states approach $\sim0.005e^2/\epsilon\ell$, which is roughly an order of magnitude smaller than the corresponding gap values for the strongest FQHE states ($\nu=1/3$, 2/3) in the LLL. Comparison of the energy gaps for samples and *A* and *B* shows that the gaps for the $\nu=5/2$, 7/3, and 8/3 states are, respectively, reduced by approximately 50%, 60%, and 70% with increasing disorder. We note that disorder apparently affects the odd-denominator states more strongly than the $\nu=5/2$ state. This behavior in sample *B* is similar to that of Ref. 6, confirming the sensitivity of FQHE states in the SLL to disorder. The sample structure used in Ref. 6 is a single-sided doped heterostructure, not the symmetrically doped quantum well in the samples we studied. This may account for the apparent reduction in the energy gap in Ref. 6 even though the sample of Ref. 6 has a higher mobility than our sample *B*.

One of the more intriguing features of the data is that the

TABLE I. Mobility, density, and energy gaps measured for the fractional quantum Hall states in the second Landau level.

Sample	μ [$cm^2/(V s)$]	n [cm^{-2}]	ν:	14/5	19/7	8/3	5/2	7/3	16/7	11/5
A	28.3×10^6	3.2×10^{11}	Δ (mK)	252	$\leqslant$110	562	544	584	$\leqslant$100	160
			$\Delta\left(\frac{e^2}{\epsilon\ell}\right)$	0.0023		0.0050	0.0047	0.0049		0.0013
B	10.5×10^6	2.8×10^{11}	Δ (mK)	$\leqslant$60		$\leqslant$150	272	206		$\leqslant$40
			$\Delta\left(\frac{e^2}{\epsilon\ell}\right)$			$\leqslant$0.0014	0.0026	0.0019		
Ref. 6	17×10^6	2.3×10^{11}	Δ (mK)			55	110	100		
			$\Delta\left(\frac{e^2}{\epsilon\ell}\right)$			0.0006	0.0012	0.0010		

energy gaps of the $\nu=7/3$ and $8/3$ states in the clean limit are disproportionally larger than what may be expected under the standard model of FQHE in the LLL. The LLL energy gaps for the $\nu=2/5$ and $3/5$ states are approximately half of the gaps for the $\nu=1/3$ and $2/3$ states.[16] In contrast, the gaps for the $\nu=7/3$ and $8/3$ states are approximately 8–10 times larger than the $\nu=12/5$ and $13/5$ states. This means that measured energy gaps for the $\nu=7/3$ and $8/3$ states are anomalously enhanced compared to what may be expected under the composite fermion model or equivalently the gaps for 12/5 and 13/5 are anomalously suppressed.

Our finding that the strongest SLL incompressible states are 5/2, 7/3, 8/3, 14/5, and 11/5 should be contrasted with the corresponding LLL incompressible states 1/3, 2/3, 2/5, 3/5, and 1/5. In the LLL, the hierarchy states (i.e., 2/5, 3/5, etc.) are the strongest fractions after the 1/3 state with $\Delta_{1/3}/\Delta_{1/5}\sim10$ (Ref. 17) and $\Delta_{1/3}/\Delta_{2/5}\sim2$ (Ref. 16). We find $\Delta_{7/3}/\Delta_{11/5}\sim4$ whereas $\Delta_{7/3}/\Delta_{12/5}\sim10$ using the activation gap of 70 mK quoted in Ref. 7 for the 12/5 state. The sample from Ref. 7 is comparable with our sample *A* in density ($n=3\times10^{11}$ cm^{-2}), mobility [$\mu=31\times10^6$ $cm^2/(V s)$], and the width of quantum well

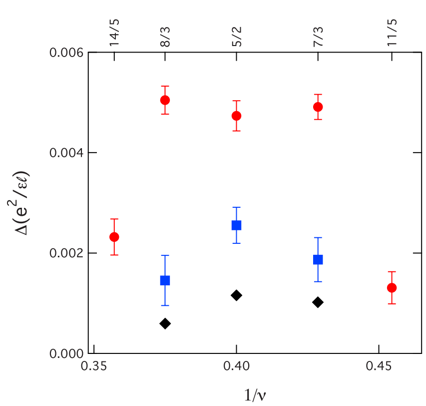

FIG. 3. (Color online) Energy gaps for the fractional quantum Hall effect states in the second Landau level in the units of Coulomb energy $e^2/\epsilon\ell$, where $\epsilon=13.1$ is the dielectric constant and $\ell=\sqrt{\hbar/eB}$ is the magnetic length. Solid dots (squares) represent the energy gaps from sample *A* (*B*). Diamonds represent energy gaps from Ref. 6.

(30 nm), but is "too weak" compared with the 11/5 and 14/5 states, exactly the reverse of the situation in the LLL. This is consistent with theoretical predictions that the Laughlin state at $\nu=2+1/3$ is unstable whereas the $\nu=2+1/5$ state is stable.[14] Based on the large gap of 7/3 and 8/3 relative to the gaps of 11/5 and 14/5 states, we thus conclude that the 7/3 and 8/3 states are unlikely to be the SLL analogs of the 1/3 and 2/3 LLL Laughlin states whereas our observed 11/5 and 14/5 states are likely to be Laughlin states. It seems, therefore, that the SLL correlations are much more subtle than the LLL correlations.[18]

The best current theoretical estimate for the infinite system extrapolated excitation gap for the 5/2 incompressible state is $\Delta_{ex}\approx0.025$ in the Coulomb energy unit.[19,20] Comparison with experiment requires (at least) three corrections due to the finite width[19] of the quasi-2D system, disorder,[19] and Landau-level mixing[21] to the ideal gap. The finite-width correction depends[19] on the parameter w/ℓ where w is the quantum well width ($w=30$ nm for both samples *A* and *B*). Using the applied magnetic field values (5.3 T for *A* and 4.6 T for *B*) we find $w/\ell\approx2.7$ (sample *A*) and 2.50 (sample *B*). Such large values of w/ℓ imply rather strong finite width corrections[19] reducing Δ_{ex} at $\nu=5/2$ by a factor of 2 or more to about 0.013, which corresponds to a gap of 1.5 K (sample *A*) and 1.4 K (sample *B*). Our observed 5/2 activation gaps $\Delta=0.54$ K (sample *A*) and 0.27 K (sample *B*) are substantially below the ideal gap values because of disorder (and, possibly, Landau-level mixing), effects which are difficult to treat theoretically. We ignore Landau-level coupling effects, although it may very well not be negligible in reality, based on the argument that $(e^2/\epsilon\ell_1)/\hbar\omega_c\approx0.4$ is small, where $\ell_1=\sqrt{(2n+1)}\ell$ is the Landau radius in the $n=1$ Landau level and ω_c is the cyclotron frequency.

The inclusion of disorder in the theory of the FQHE gap is problematic in the absence of a true transport theory. A simple procedure, used extensively if somewhat unjustifiably, is to write the disorder-induced gap as $\Delta\equiv\Delta_{ex}-\Gamma$ where Γ is the calculated level broadening. We can theoretically estimate the zero-field level broadening of samples *A* and *B* by using the sample structures ($w=30$ nm with a spacer layer of $d=80$ nm) and the mobilities to get $\Gamma_A\approx0.76$ K and $\Gamma_B\approx1.26$ K, where the level broadening Γ

corresponds to the so-called quantum single-particle impurity broadening ($\Gamma_s \equiv \hbar/2\tau_s$) rather than the transport mobility broadening ($\Gamma_t \equiv \hbar/2\tau_t$). It is well known[22] that in high-mobility modulation-doped structures $\tau_t/\tau_s \gg 1$, and in fact, for the high-mobility structures used in our experiments $\Gamma_s \approx 200\Gamma_t$ due to the very large values of $q_s d \sim 20$, where q_s is the screening wave vector. In addition, our estimated Γ_s's for our two samples are consistent with the low magnetic field onset ($\leqslant 0.01$ T) of Shubnikov-de Haas oscillations corresponding to the $\omega_c/\Gamma_s \sim 1$ condition.

Incorporating a disorder (and finite-width) correction into the theoretical gap values we arrive at the following predictions for the activation gaps: $\Delta_A = 1.5 - 0.8$ K ≈ 0.7 K and $\Delta_B = 1.4 - 1.2$ K ≈ 0.2 K, which are comparable with our experimentally measured gaps of 0.54 and 0.27 K, respectively. We note that much of the suppression (a factor of 2) of the measured 5/2 gap (~ 0.005) compared with the theoretical 5/2 gap (~ 0.025) arises from the large effective well width value ($\omega/\ell \sim 3$) in our sample, which differs somewhat from earlier theoretical works in the literature where disorder broadening[19] or Landau-level mixing[21] were taken to be the dominant mechanisms suppressing the experimental gap. We also note that further improvement [above the $\mu_A = 28 \times 10^6$ cm^2/(V s) s value of sample A] in the sample quality could enhance the gap at most by a factor of 2 provided mobilities above 50×10^6 could be achieved. Our theoretical consideration actually suggests two alternative (and perhaps simpler) techniques for enhancing the 5/2 gap: (i) use thinner quantum well samples so that the finite width correction is smaller and (ii) use a higher carrier density so that the 5/2 FQHE state occurs at higher magnetic field values.

Based on our extensive FQHE activation measurements in the second Landau level, we conclude that (i) the even-denominator 5/2 state now possesses an energy gap that exceeds 500 mK, (ii) the 5/2 state, which has no analog in the LLL, is the most robust state against disorder in the SLL; and (iii) the 2+1/3 (and the related 2+2/3) SLL states are unlikely to be Laughlin-like states similar to the corresponding 1/3 or 2/3 states in the LLL. Our measured activation energy for the 7/3 state is an order of magnitude larger than the 12/5 activation energy, but is within a factor of 4 of the 11/5 activation energy. For the LLL-Laughlin-like states the situation is precisely reversed with the 1/3 state having an activation energy an order of magnitude (only a factor of 4) larger than the 1/5 (2/5) state. The SLL, in contrast to the LLL where the Laughlin correlation dominates except at the smallest filling factors, possesses many competing ground states of comparable energies for all fillings, considerably complicating the task of understanding its unique and rich quantum phase diagram. Our measured SLL activation gaps are by far the largest ever reported for 5/2, 7/3, 8/3, 11/5, and 14/5 although our measurement temperature is a relatively modest 36 mK. Given the great deal of current theoretical interest in the SLL FQHE,[13,20,21,23,24] we believe that our results could lead to a better understanding of the SLL FQHE.

We would also like to note that we recently learned of a paper[25] by Pan *et al.* reporting on related findings.

We thank R.H. Morf for careful reading of the manuscript, W. Pan for useful discussions, and the University of Chicago MRSEC for the use of shared facilities. This work is supported by the Microsoft Q Project.

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